Scaling Properties of Urban Radiance

Christopher Small
Lamont Doherty Earth Observatory
Columbia University

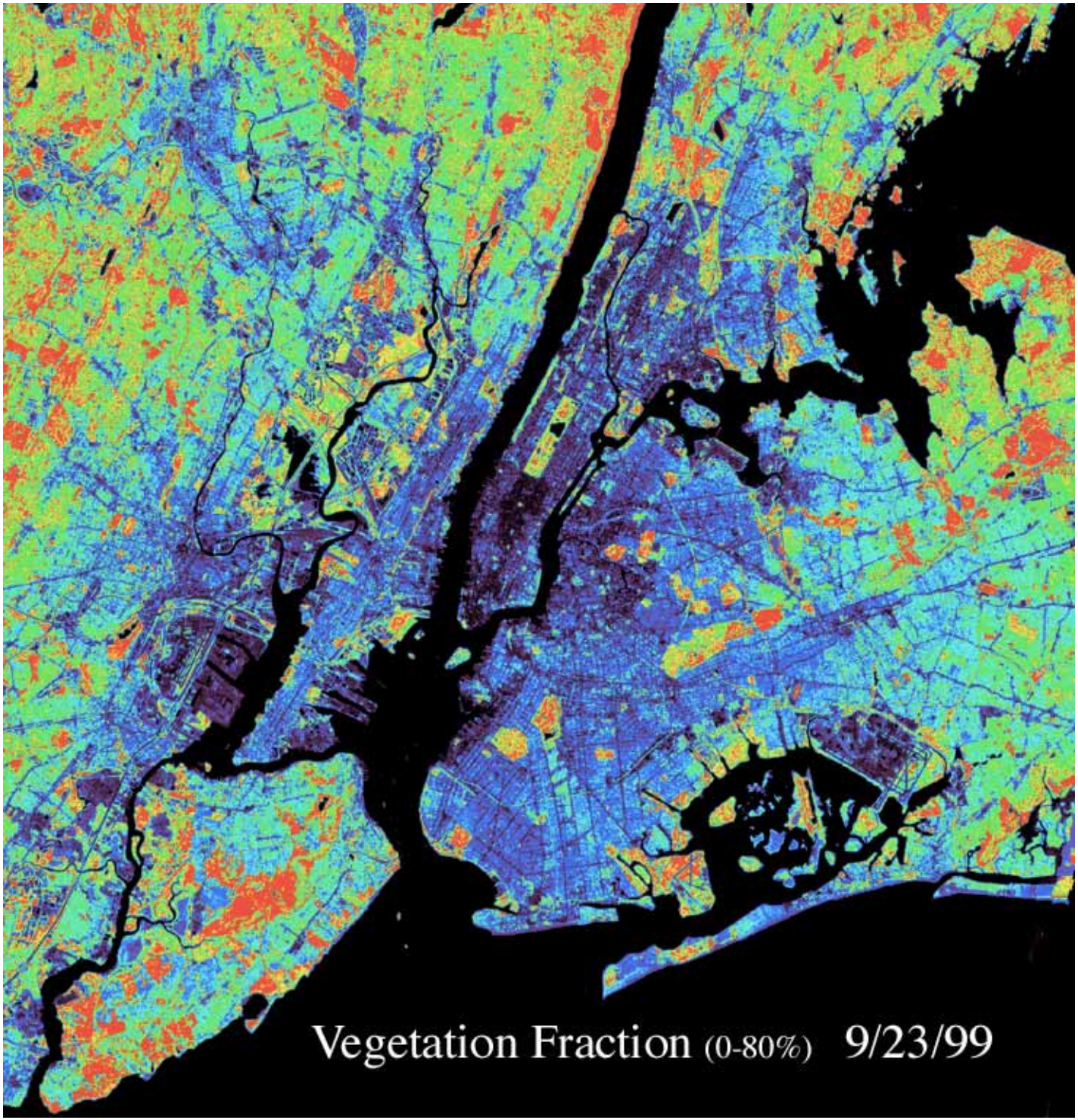
Research

Energy/Mass Flux Scaling
Urban Vegetation Abundance
Aerosol Distribution

Application

Urban Microclimate
Urban Ecology
Air Quality

Landsat 7 ETM+ (RGB=7/4/2) 9/23/99


Vegetation Fraction (0-80%) 9/23/99

A Heirarchy of Questions

Urban Vegetation Abundance & Spatial Distribution - *How does it affect energy consumption & air quality?*

Spectral Mixing & Fine Scale Vegetation Mapping - *How Linear? How Accurate?*

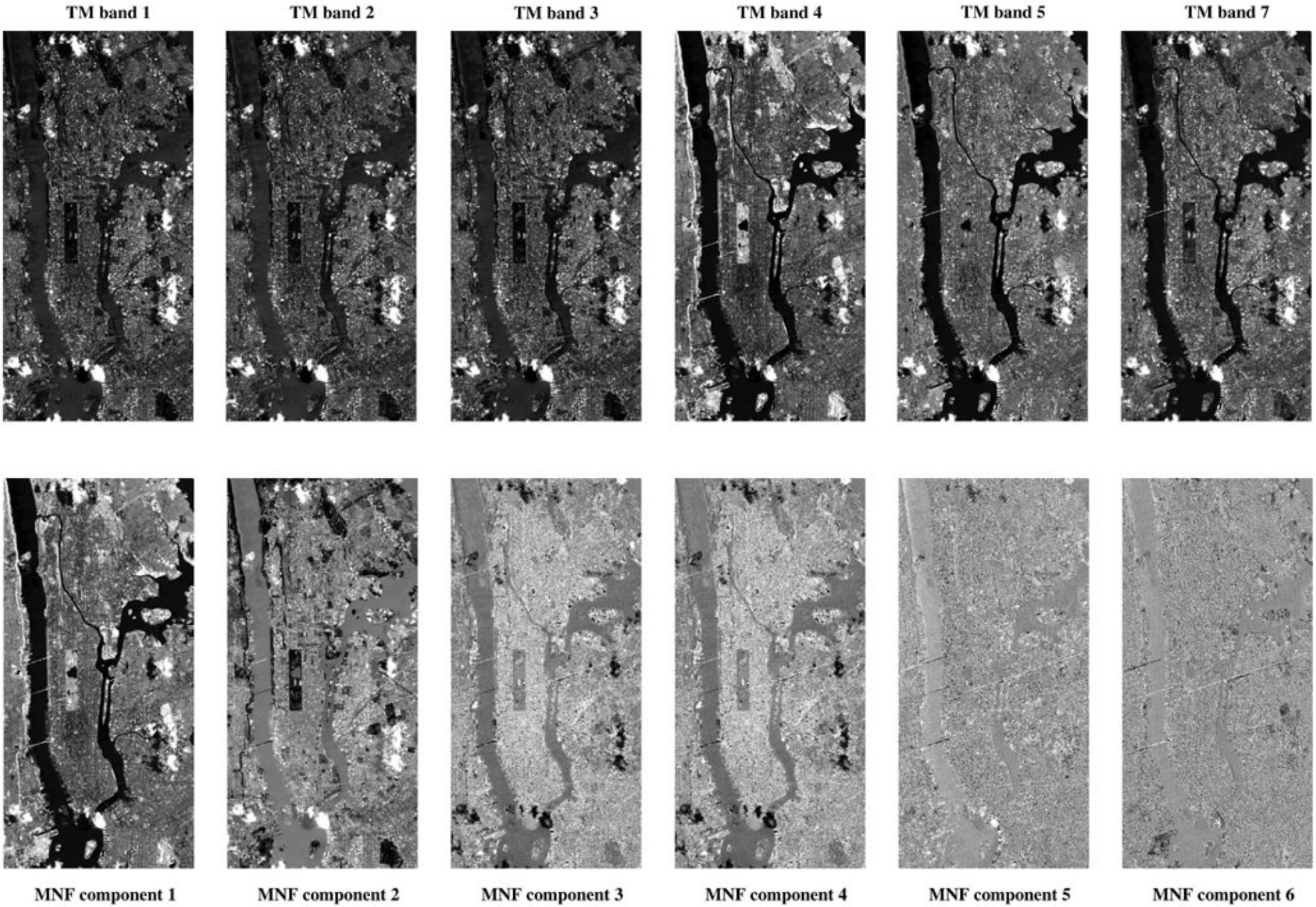
Scale Dependence & Spectral Dimensionality - *What is the tradeoff?*

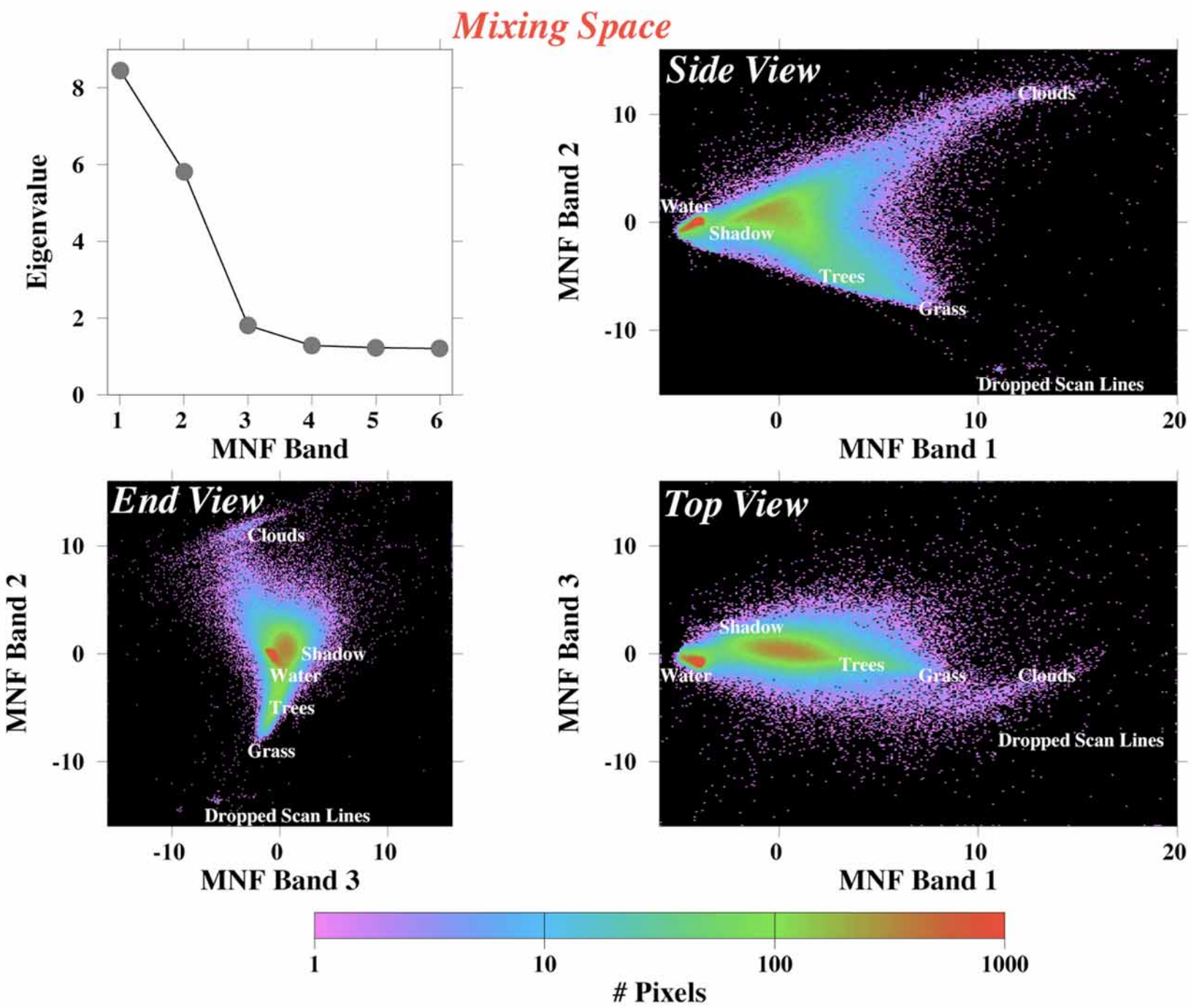
Signal/Noise & Spectral Resolution - *Effect on apparent spectral resolution?*

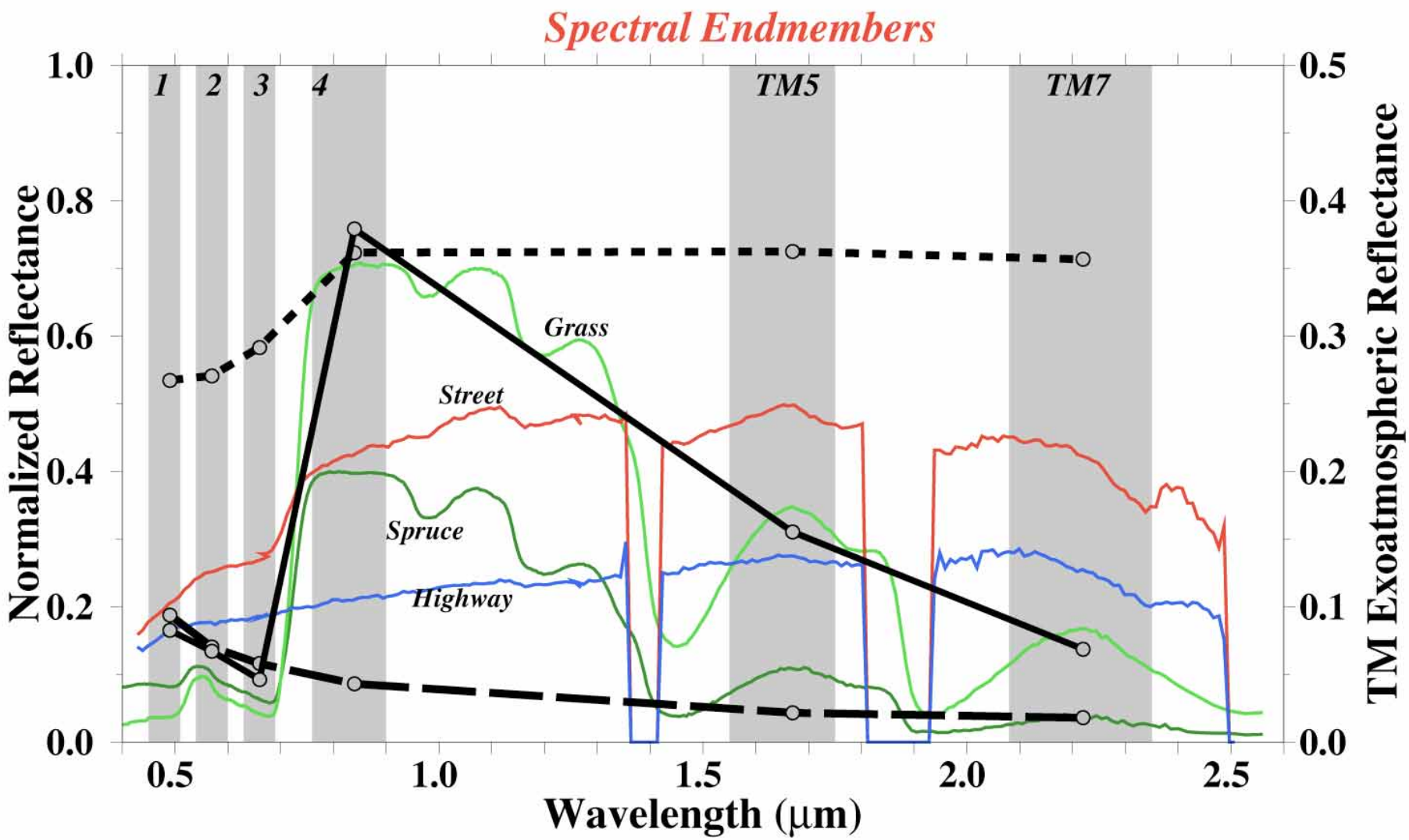
The questions will be addressed in an analysis of 7 U.S. Cities.

The objective is to quantify and analyse abundance and distribution of Urban & Suburban vegetation.

Atlanta, Chicago, Houston, Los Angeles, New York, Phoenix, Seattle.


TM band 1
TM band 2
TM band 3
TM band 4
TM band 5
TM band 7
MNF component 1
MNF component 2
MNF component 3
MNF component 4
MNF component 5
MNF component 6


Mixing Space
Eigenvalue
MNF Band
Side View
MNF Band 2
MNF Band 1
Clouds
Water
Shadow
Trees
Grass
Dropped Scan Lines
End View
MNF Band 2
MNF Band 3
Clouds
Shadow
Water
Trees
Grass
Dropped Scan Lines
Top View
MNF Band 3
MNF Band 1
Shadow
Water
Trees
Grass
Clouds
Dropped Scan Lines
Pixels


Spectral Endmembers
1
2
3
4
TM5
TM7
Grass
Street
Spruce
Highway
Normalized Reflectance
TM Exoatmospheric Reflectance
Wavelength (μm)
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.4
0.3
0.2
0.1
0.0
0.5
1.0
1.5
2.0
2.5

Spectral reflectance can be described as a linear combination of endmember spectra as:

$$f_1\mathbf{E}_1(\lambda) + f_2\mathbf{E}_2(\lambda) \ldots + f_n\mathbf{E}_n(\lambda) = \mathbf{R}(\lambda)$$

$\mathbf{R}(\lambda)$ is the observed reflectance profile, a continuous function of wavelength λ.
$\mathbf{E}_i(\lambda)$ are the endmember spectra and
f_i are the corresponding fractions of the n endmembers

Continuous reflectance profiles are represented as vectors of discrete reflectance estimates at specific wavelengths as:

$$\mathbf{E}(\lambda) = [e_{\lambda 1}, e_{\lambda 2} \ldots e_{\lambda n}] \quad \text{and} \quad \mathbf{R}(\lambda) = [r_{\lambda 1}, r_{\lambda 2} \ldots r_{\lambda n}]$$

$r_{\lambda i}$ represents a portion of the observed reflectance spectrum
$\mathbf{R}(\lambda)$, integrated over a finite spectral band with a center wavelength λ_i and
$e_{\lambda i}$ represents observed reflectance from the corresponding endmember $\mathbf{E}(\lambda)$.

The continuous linear mixing model can be represented in discrete form as a system of linear mixing equations

$$f_j\mathbf{e}_{ij} = \mathbf{r}_i \qquad i = 1,b \quad \text{and} \quad j = 1,n$$

The system of b linear equations can be written as:

$$\mathbf{Ef} = \mathbf{r}$$

The overdetermined linear mixing model, incorporating measurement error:

$$\mathbf{r} = \mathbf{Ef} + \varepsilon$$

ε is an error vector which must be minimized to find the fraction vector $\mathbf{f}$ which gives the best fit to the observed reflectance vector $\mathbf{r}$. Since $\varepsilon = \mathbf{r} - \mathbf{Ef}$, we seek to minimize:

$$\varepsilon^T\varepsilon = (\mathbf{r} - \mathbf{Ef})(\mathbf{r} - \mathbf{Ef}).$$

In the case of uncorrelated noise, the well known least squares solution is given by:

$$\mathbf{f} = (\mathbf{E}^T\mathbf{E})^{-1}\mathbf{E}^T\mathbf{r}$$

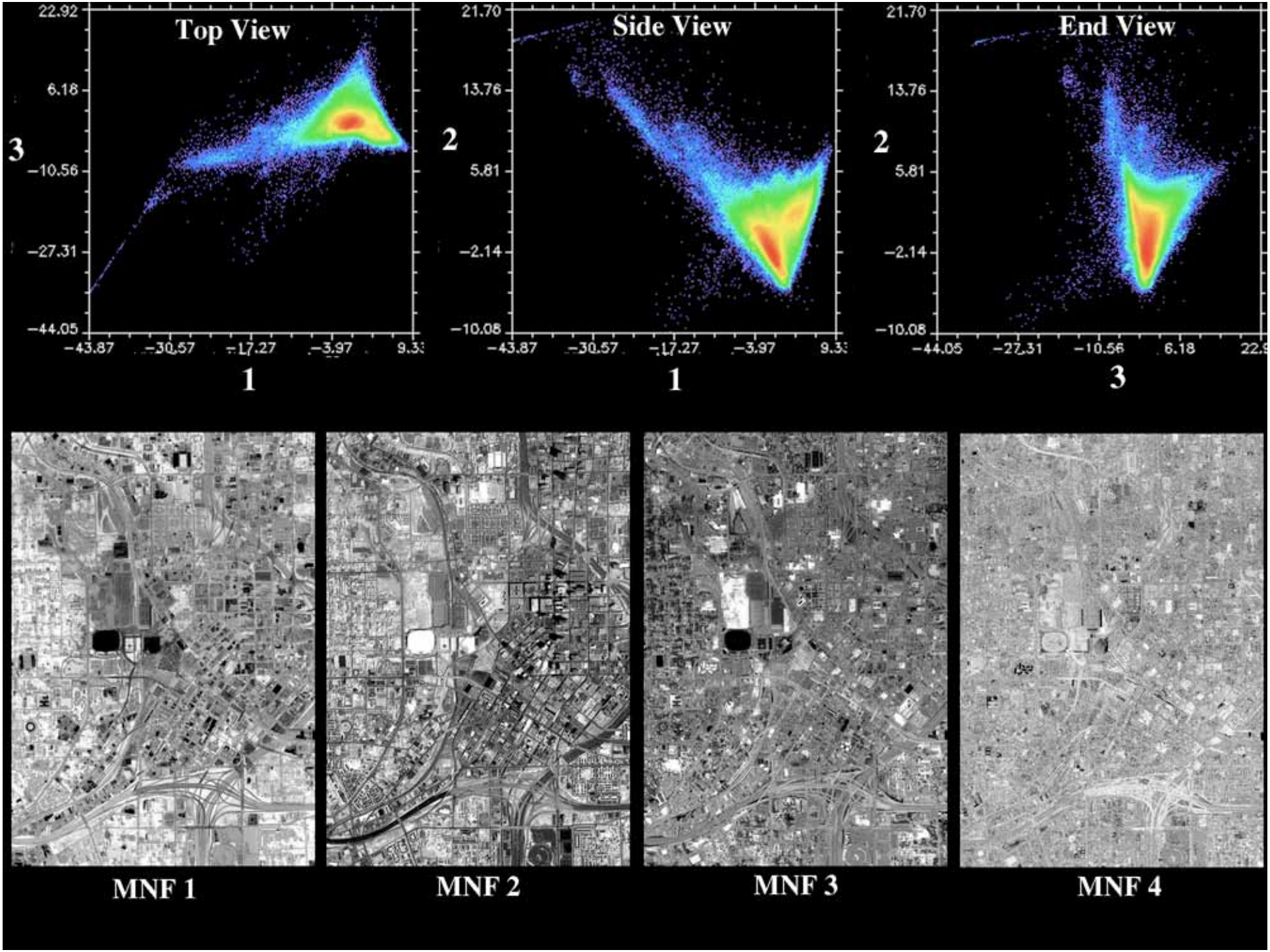
3 Endmember Linear Mixing Model

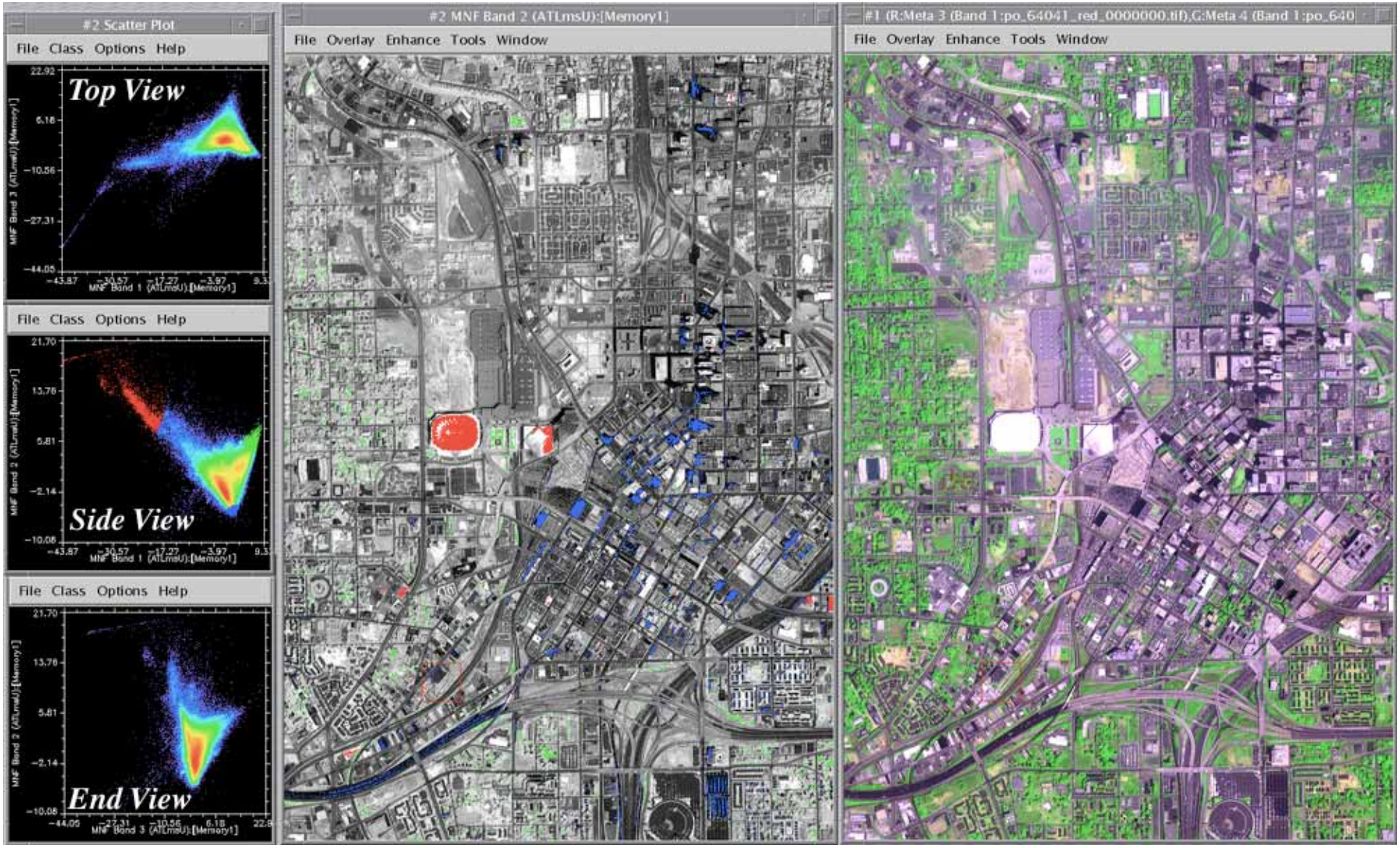
$$
\begin{array}{ccccccc}
f_1 E_{1\lambda 1} & & f_2 E_{2\lambda 1} & & f_3 E_{3\lambda 1} & & R_{\lambda 1} \\
f_1 E_{1\lambda 2} & & f_2 E_{2\lambda 2} & & f_3 E_{3\lambda 2} & & R_{\lambda 2} \\
f_1 E_{1\lambda 3} & + & f_2 E_{2\lambda 3} & + & f_3 E_{3\lambda 3} & = & R_{\lambda 1} \\
f_1 E_{1\lambda 4} & & f_2 E_{2\lambda 4} & & f_3 E_{3\lambda 4} & & R_{\lambda 4} \\
f_1 E_{1\lambda 5} & & f_2 E_{2\lambda 5} & & f_3 E_{3\lambda 5} & & R_{\lambda 5} \\
f_1 E_{1\lambda 6} & & f_2 E_{2\lambda 6} & & f_3 E_{3\lambda 6} & & R_{\lambda 6}
\end{array}
$$

To first order, radiances mix linearly in proportion to area.

Given some knowledge of the spectral endmembers (E), it is possible to* estimate *fractions (f) contributing to a spectrally mixed radiance measurement (R).

~~***SubPixel Resolution***~~

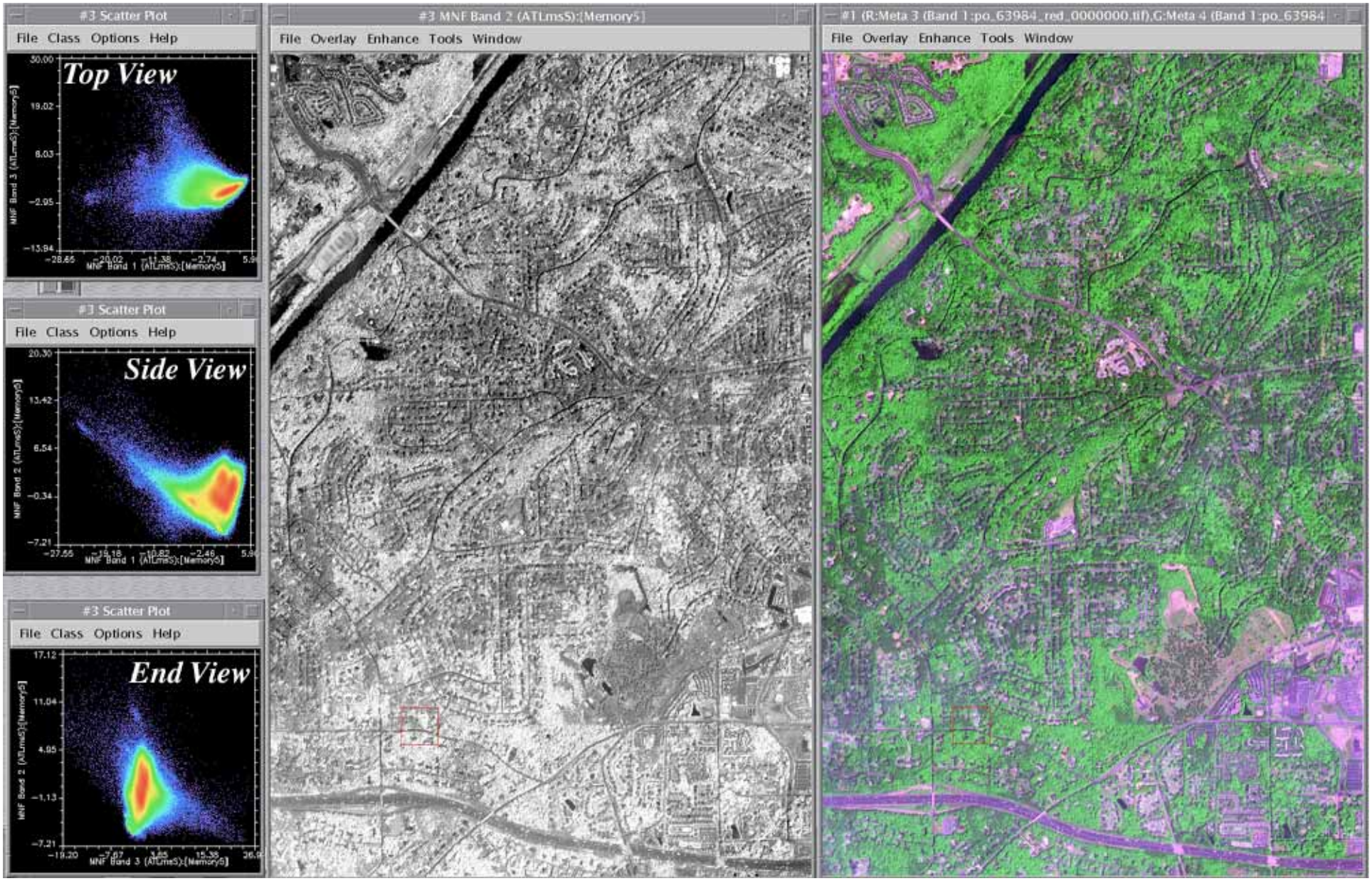

Top View
Side View
End View
22.92
6.18
-10.56
-27.31
-44.05
21.70
13.76
5.81
-2.14
-10.08
-43.87
-30.57
-17.27
-3.97
9.33
-44.05
-27.31
-10.56
6.18
22.9
1
2
3
MNF 1
MNF 2
MNF 3
MNF 4

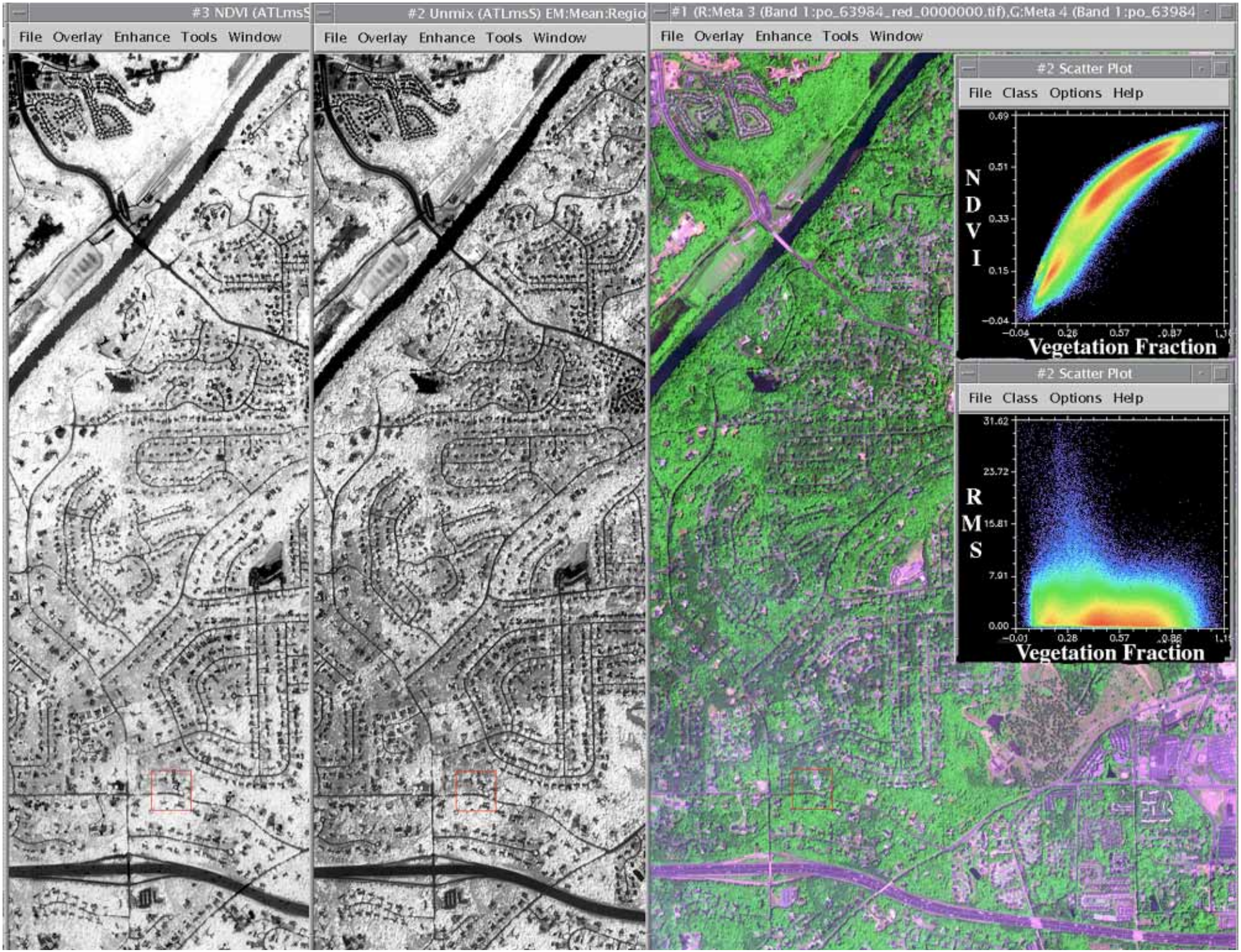

#2 Scatter Plot
File Class Options Help
Top View
MNF Band 3 (ATLmsU):[Memory1]
MNF Band 1 (ATLmsU):[Memory1]
File Class Options Help
Side View
MNF Band 2 (ATLmsU):[Memory1]
MNF Band 1 (ATLmsU):[Memory1]
File Class Options Help
End View
MNF Band 2 (ATLmsU):[Memory1]
MNF Band 3 (ATLmsU):[Memory1]
#2 MNF Band 2 (ATLmsU):[Memory1]
File Overlay Enhance Tools Window
#1 (R:Meta 3 (Band 1:po_64041_red_0000000.tif),G:Meta 4 (Band 1:po_640
File Overlay Enhance Tools Window

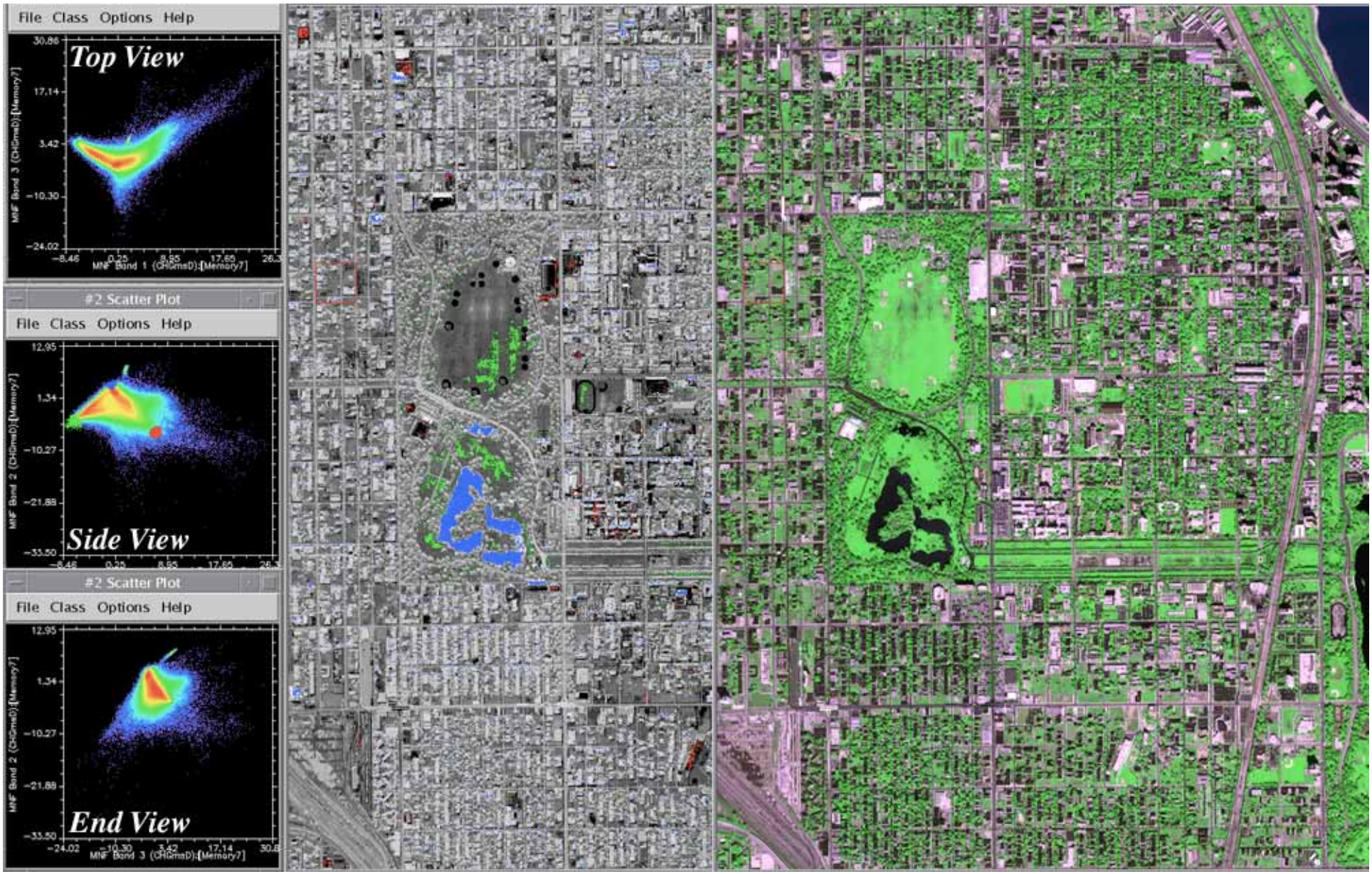
Atlanta GA, 5/18/2000

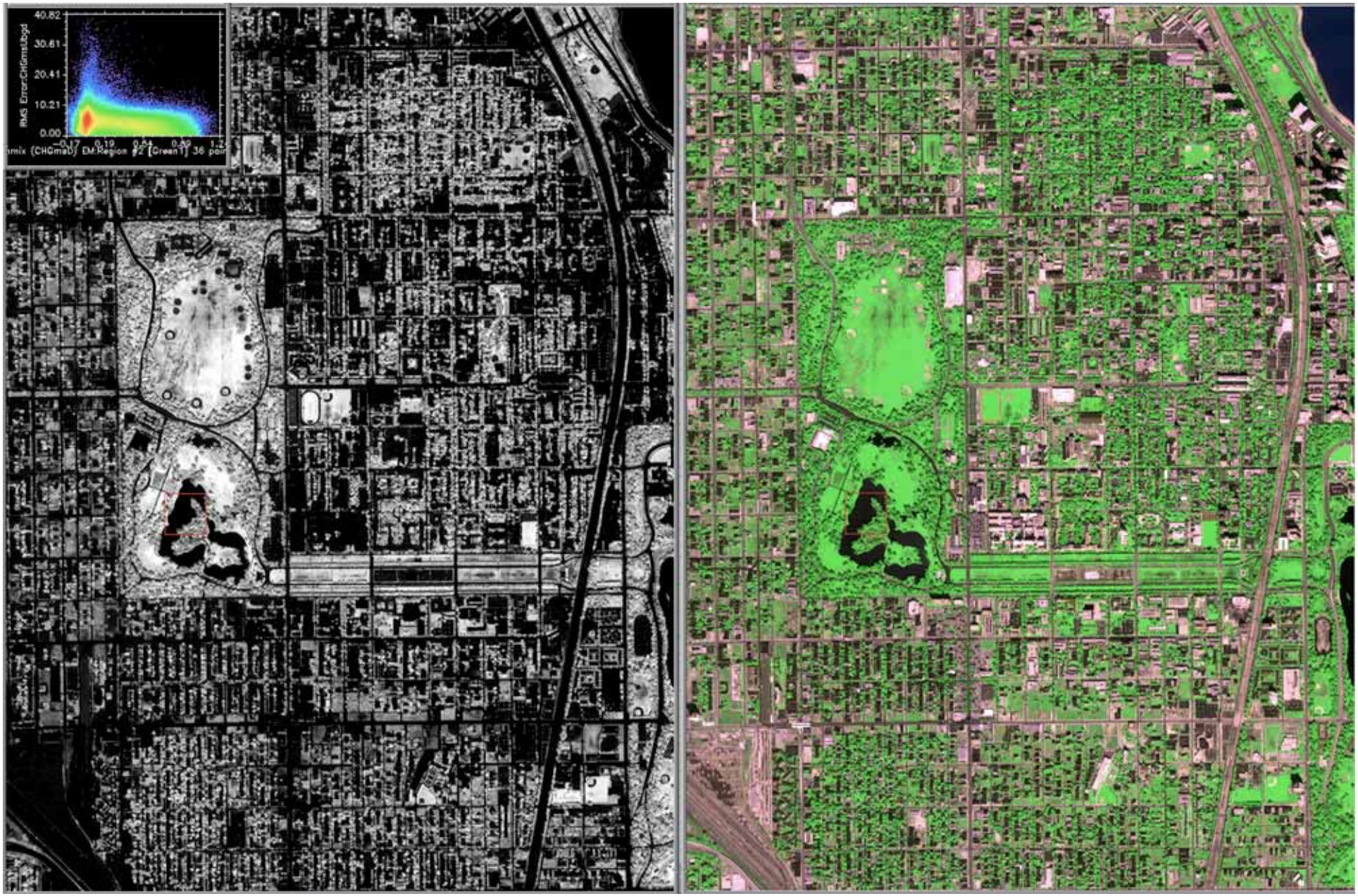
Ikonos (RGB=341)

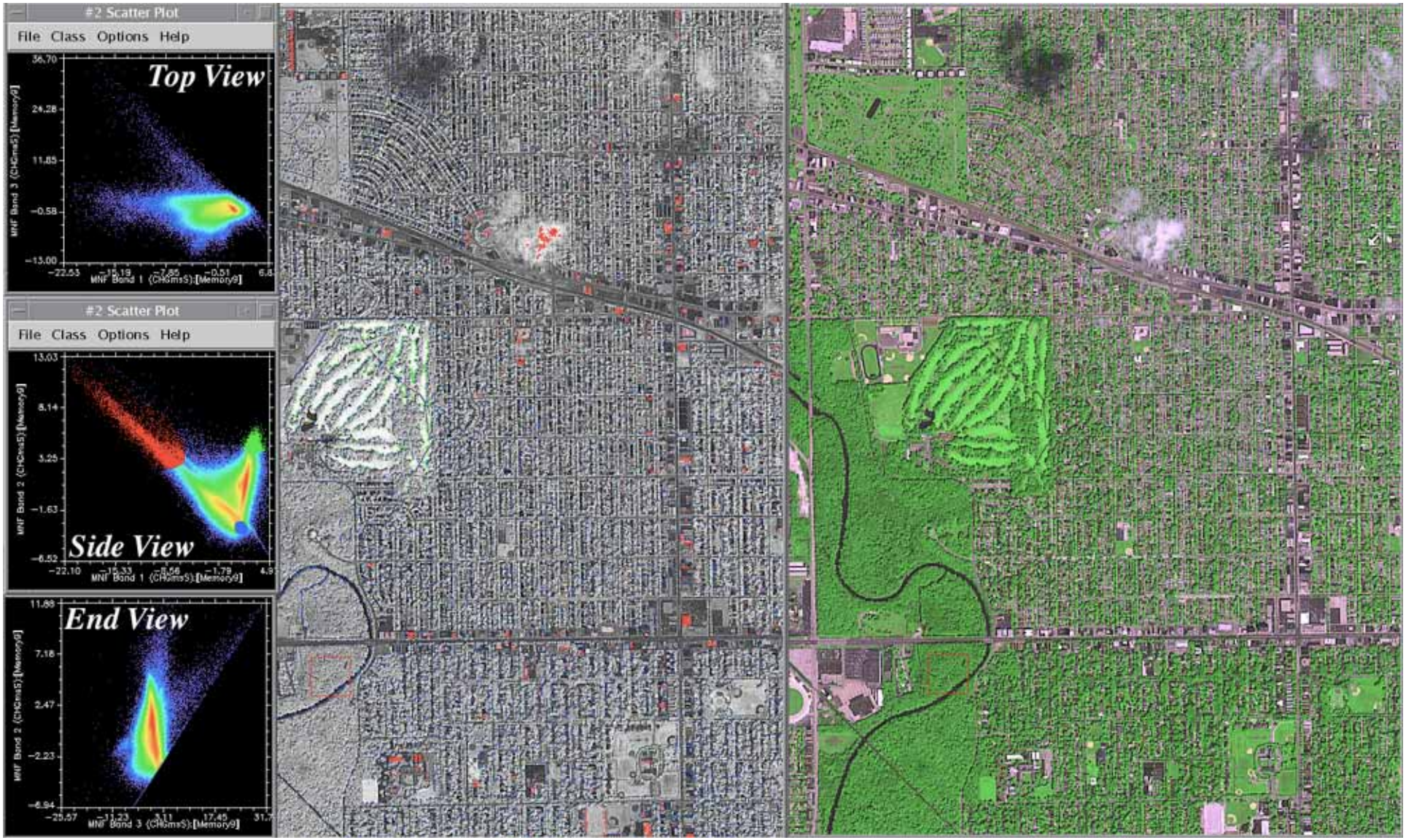
Vegetation Fraction


#3 Scatter Plot
File Class Options Help
Top View
#3 Scatter Plot
File Class Options Help
Side View
#3 Scatter Plot
File Class Options Help
End View
#3 MNF Band 2 (ATLmsS):[Memory5]
File Overlay Enhance Tools Window
#1 (R:Meta 3 (Band 1:po_63984_red_0000000.tif),G:Meta 4 (Band 1:po_63984
File Overlay Enhance Tools Window

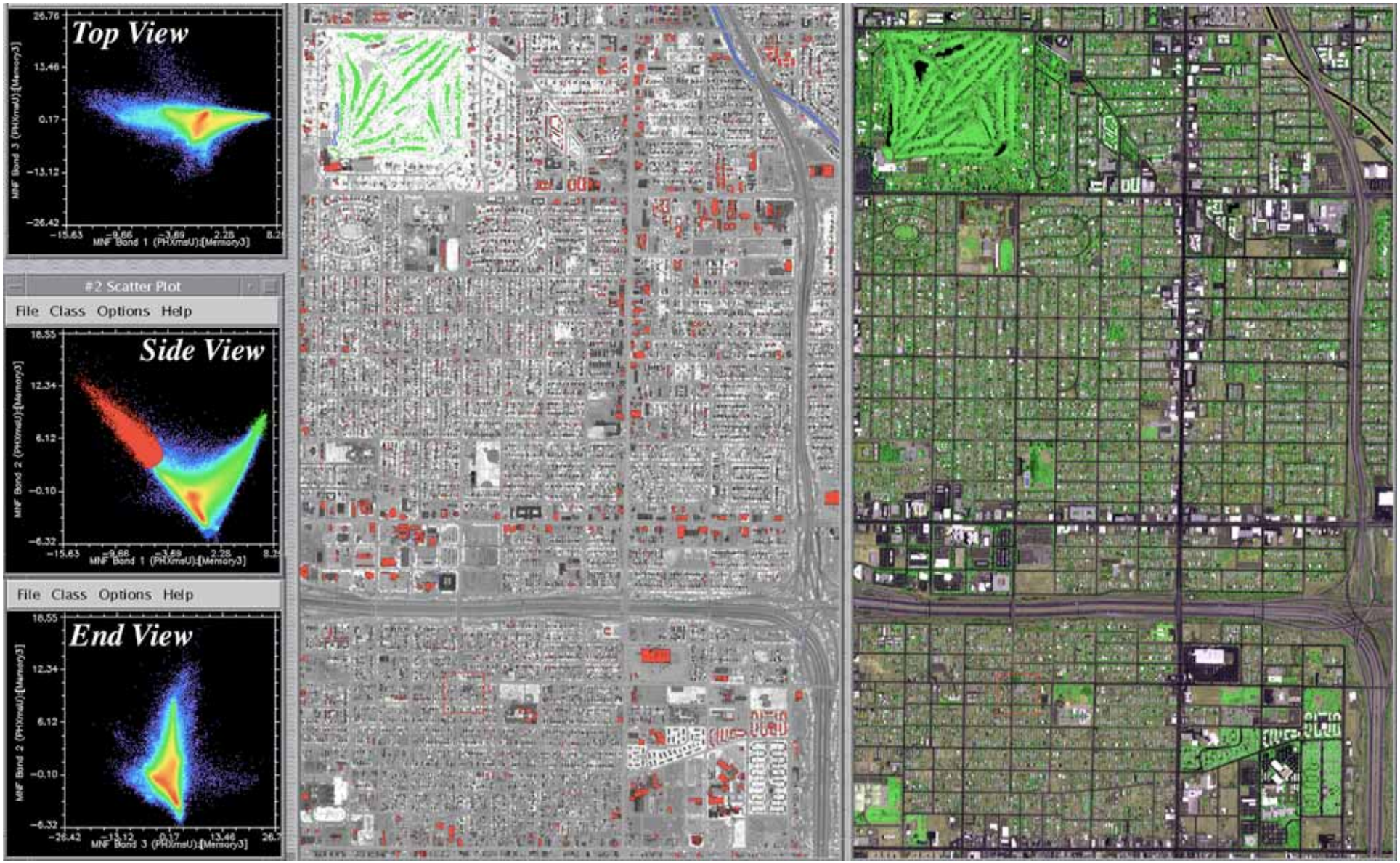

#3 NDVI (ATLmsS
File Overlay Enhance Tools Window
#2 Unmix (ATLmsS) EM:Mean:Regio
File Overlay Enhance Tools Window
#1 (R:Meta 3 (Band 1:po_63984_red_0000000.tif),G:Meta 4 (Band 1:po_63984
File Overlay Enhance Tools Window
#2 Scatter Plot
File Class Options Help
0.69
0.51
0.33
0.15
-0.04
NDVI
-0.04
0.26
0.57
0.87
Vegetation Fraction
#2 Scatter Plot
File Class Options Help
31.62
23.72
15.81
7.91
0.00
RMS
-0.01
0.28
0.57
0.86
Vegetation Fraction

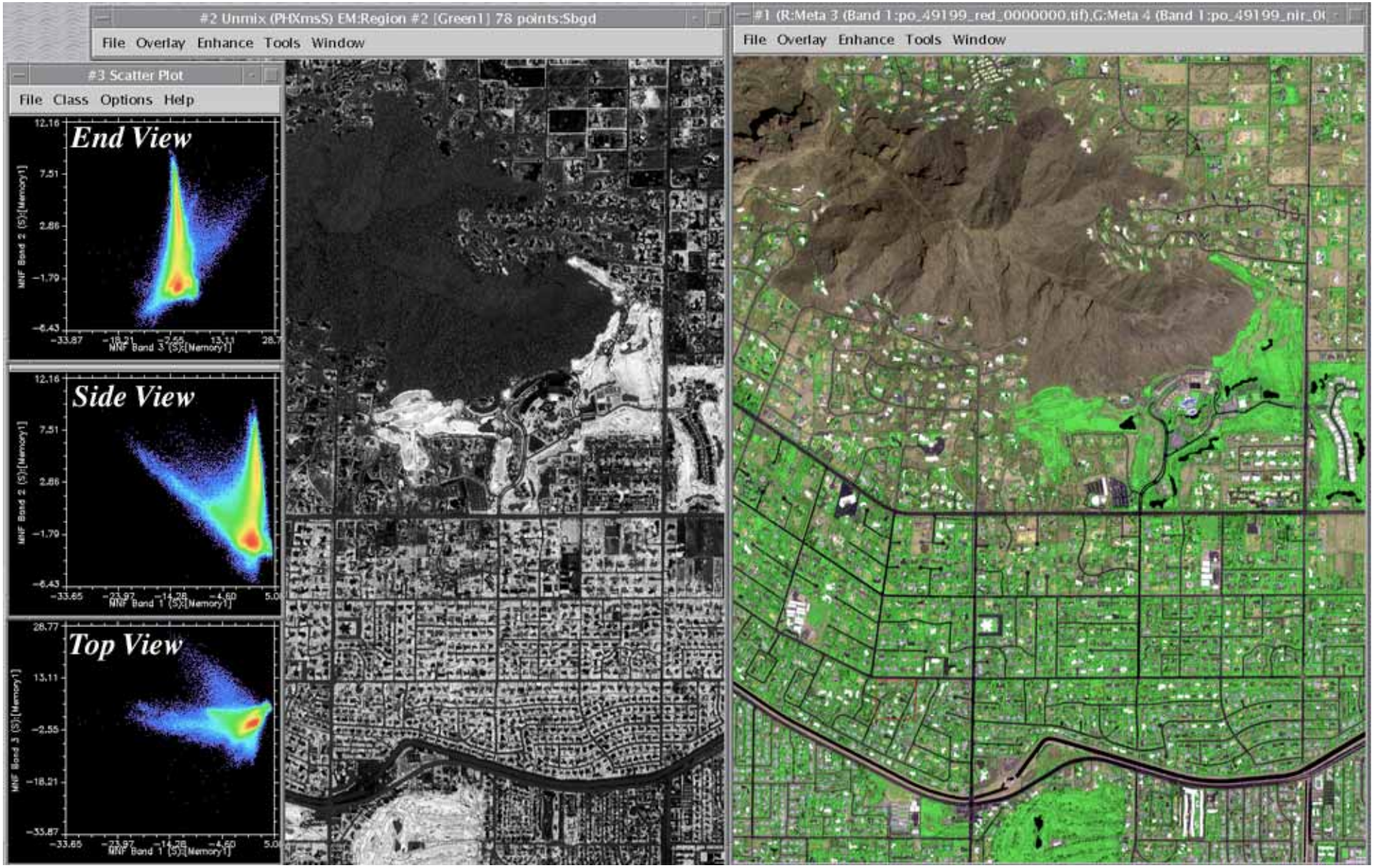

File Class Options Help
Top View
30.86
17.14
3.42
−10.30
−24.02
−8.46
0.25
8.85
17.65
MNF Band 1 (CHGmsD):[Memory7]
MNF Band 3 (CHGmsD):[Memory7]
#2 Scatter Plot
File Class Options Help
12.95
1.34
−10.27
−21.88
−33.50
Side View
MNF Band 2 (CHGmsD):[Memory7]
#2 Scatter Plot
File Class Options Help
End View
−24.02
−10.30
3.42
17.14
MNF Band 3 (CHGmsD):[Memory7]

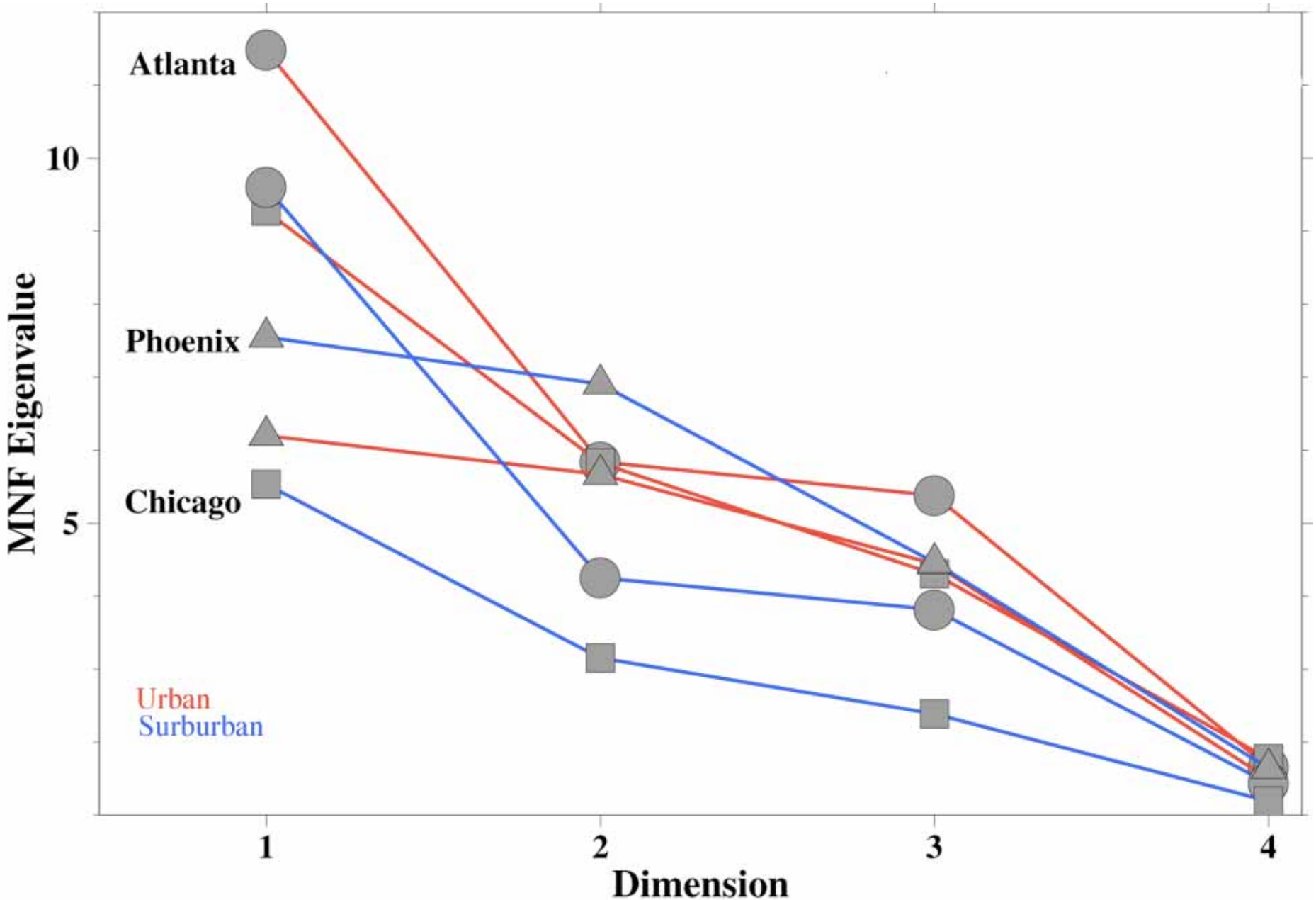

40.82
30.61
20.41
10.21
0.00
RMS Error:CHGmsUbgd
-0.17
0.19
0.54
0.89
1.2
mix (CHGmsD) EM:Region #2 [Green1] 36 poi


#2 Scatter Plot
File Class Options Help
Top View
36.70
24.28
11.85
-0.58
-13.00
-22.53
-15.19
-7.85
-0.51
MNF Band 1 (CHGmsS):[Memory9]
MNF Band 3 (CHGmsS):[Memory9]
#2 Scatter Plot
File Class Options Help
Side View
13.03
8.14
3.25
-1.63
-6.53
-22.10
-15.33
-8.56
-1.79
MNF Band 1 (CHGmsS):[Memory9]
MNF Band 2 (CHGmsS):[Memory9]
End View
11.88
7.18
2.47
-2.23
-6.94
-25.57
-11.23
3.11
17.45
MNF Band 3 (CHGmsS):[Memory9]
MNF Band 2 (CHGmsS):[Memory9]

94.36
70.77
47.18
23.59
0.00
RMS Error:[Memory12]
-0.49
-0.00
0.48
0.97
1.44
mix (CHGmtS) EM:Region #2 [Green1] 30 points


Top View
26.76
13.46
0.17
−13.12
−26.42
MNF Band 3 (PHXmsU):[Memory3]
−15.63
−9.66
−3.69
2.28
MNF Band 1 (PHXmsU):[Memory3]
#2 Scatter Plot
File Class Options Help
Side View
18.55
12.34
6.12
−0.10
−6.32
MNF Band 2 (PHXmsU):[Memory3]
File Class Options Help
End View
−26.42
−13.12
0.17
13.46
MNF Band 3 (PHXmsU):[Memory3]


#2 Unmix (PHXmsS) EM:Region #2 [Green1] 78 points:Sbgd
File Overlay Enhance Tools Window
#3 Scatter Plot
File Class Options Help
End View
Side View
Top View
MNF Band 2 (S):[Memory1]
MNF Band 3 (S):[Memory1]
MNF Band 1 (S):[Memory1]
#1 (R:Meta 3 (Band 1:po_49199_red_0000000.tif),G:Meta 4 (Band 1:po_49199_nir_0
File Overlay Enhance Tools Window


Atlanta
Phoenix
Chicago
Urban
Surburban
10
5
MNF Eigenvalue
1
2
3
4
Dimension

Conclusions

Apparent dimensionality of Ikonos MSI imagery exceeds noise threshold.

Urban & Suburban areas analysed show3+ dimensional mixing spaces with significant nonlinearity.

The 3 endmember linear mixing model produces < 5% RMS misfit for vegetation component.

Urban & Suburban areas have similar eigenvalue distributions but distinct mixing spaces.